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ON THE ANALOGY BETWEEN THERMALLY AND IRRADIATION INDUCED CREEP. (U)
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Department of Engineering Science
Aerospace Engineering and Nuclear Engineering
FACULTY OF ENGINEERING AND APPLIED SCIENCES



State University of New York at Buffalo

Report No. 96

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F. A. Cozzarelli and S. Huang



September, 1976

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DEPARTMENT OF ENGINEERING SCIENCE,
AEROSPACE ENGINEERING AND NUCLEAR ENGINEERING
FACULTY OF ENGINEERING AND APPLIED SCIENCES
STATE UNIVERSITY OF NEW YORK AT BUFFALO

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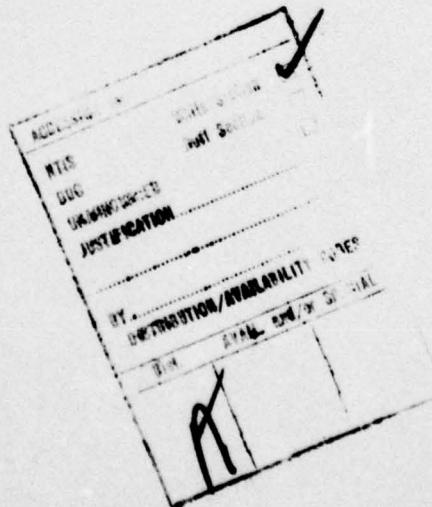
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ABSTRACT

Employing an analogy between thermally induced and irradiation induced creep, physical arguments are used first to deduce a one-dimensional constitutive relation for metals under stress in a high temperature and high neutron flux field. This constitutive relation contains modified superposition integrals in which the temperature and flux dependence of the material parameters is included via the use of two reduced time scales; linear elastic, thermal expansion and swelling terms are also included. A systematic development based on thermodynamics, with the stress, temperature increment and defect density increment as independent variables in the Gibbs free energy, is then employed to obtain general three-dimensional memory integrals for strain; the entropy and coupled energy equation are also obtained. Modified superposition integrals similar to those previously obtained by physical argument are then obtained by substituting special functions into the results of the thermodynamic analysis, and the special case of an isotropic stress power law is examined in detail.

1. INTRODUCTION

Metals under stress at elevated temperatures experience time dependent strain, i.e., the well-known phenomenon called thermally induced creep. As general rule, thermally induced creep in a metal is considered insignificant at temperatures below one third of the melting point. Similarly, metals under stress in a high neutron flux field, even at temperatures well below one third of the melting points, also experience significant creep strains. This less well-known phenomenon is called irradiation induced creep. Gilbert [1] summarized some of the possible microscopic mechanisms associated with irradiation in metals, and observed that irradiation may "enhance" thermal creep mechanisms as well as inducing new creep mechanisms. It is believed that the most significant irradiation effect in a metal is the production of point defects by the collision of high energy neutrons with the lattice atoms. Thus the production of point defects forms an essential element in any study of the enhancement of thermal creep and/or the inducement of irradiation creep.

The main purpose of this paper is to construct material constitutive relations valid in a neutron flux field at high temperatures. Since Andrade's initial work in thermal creep early in this century, many mathematical models have been proposed to represent a great accumulation of constant stress data. These models can be generalized to variable stress by using the strain or time hardening techniques (see [2]), and reasonably good results may be obtained. In [3], a single integral form of the constitutive relation is obtained by applying the strain hardening technique to each thermal creep component, and we follow this approach in the present paper. Also, since the thermal creep properties depend strongly on temperature, a thermorheologically simple material is assumed and a thermal reduced time is employed [4] in the

thermal creep compliance function.

As to irradiation induced creep, a limited amount of data has been obtained during the past two decades [5 - 9] for various metals and alloys in various geometries (e.g. rods, spring, etc.). Such data are generally for the case of constant flux and constant stress, and it is found that the creep rates are dependent on the materials, their pre-irradiation treatments, the temperature level, the stress level and the flux level. A reasonable fit for these data is obtained by a creep model characterized by the sum of linear elastic, steady creep and transient creep strain components. The strain hardening technique may be applied to generalize this type model to the case of variable stress, which may then be expressed in terms of memory integrals. The flux dependence of the material parameters in irradiation induced creep plays a role similar to the temperature dependence of the parameters in thermally induced creep, and thus we introduce the concept of a "fluxorheologically" simple material and use an irradiation reduced time in the irradiation creep compliance function. At high levels of both temperature T and flux ϕ , thermal and irradiation creep occur simultaneously (i.e. "thermoirradiation" induced creep), with the material parameters depending on both T and ϕ . For this case, Rashid [10] proposed a single memory integral with a modified reduced time containing functions of T and ϕ . In this paper we use an alternate approach whereby we superpose memory integrals with two distinct reduced times, where both times contain functions of T and ϕ with one dominated by the thermal dependence and the other by the flux dependence.

Constitutive relations are state equations in thermodynamics, and thus they must be consistent with ^{the} first and second laws. Thermodynamic theory has been applied to viscoelastic materials along basically two different

approaches: (1) The energy functional approach of Coleman [11], and (2) The state variable approach of Biot [12] and Schapery [13]. Similar results may be obtained through either approach, as discussed in [14]. Chang and Cozzarelli [15] employed the energy functional approach to obtain a single integral constitutive relation for nonlinear thermoviscoelastic materials. A similar approach was used in [16] to develop constitutive relations for thermally and irradiation induced creep; the present paper is essentially an amplification and generalization of that initial effort.

Although the neutron flux ϕ in irradiation creep produces an effect analogous to that of the temperature T in thermal creep, ϕ is an external input and not an internal thermodynamic variable as is T . However, Makin, Whapham and Minter [17] found that if annealing is neglected the rate of change of the imperfection density \dot{n} is proportional to the flux ϕ . Based on this observation, we employ the imperfection density n instead of the neutron flux ϕ in our thermodynamic development.

An important contribution to the total strain in a high radiation field is due to the swelling, which is analogous to the thermal expansion in a thermal field. Swelling is defined as the volume change due to neutron flux in the absence of applied stress. This phenomenon was discussed some time ago by Cawthorne and Fulton [18], and since then various empirical equations have been developed. In this paper, we assume that the swelling is simply proportional to the fluence ϕt , as in [19, 20]. Since ϕ has been taken here proportional to n , the swelling is thus proportional to the increment in defect density. This establishes a direct analogy between swelling and thermal expansion, which is of course proportional to the increment in temperature.

In Section 2, simple creep models deduced from one-dimensional creep

test data are generalized to the case of variable stress, temperature and flux, resulting in modified superposition integrals. In Section 3, the thermodynamic development of the general three-dimensional single integral form of the constitutive relation is discussed in detail. In Section 4, we show that the superposition integral representation in Section 2 is in fact a special case of the general representation in Section 3, obtainable by selecting a special form for the Gibbs free energy. In Section 5, one-dimensional expressions and compressibility effects are examined for the special case of isotropic stress power laws. A short summary then follows.

2. PHYSICAL PRELIMINARIES

In-reactor creep strain ϵ in metals is a complex function of stress σ , temperature T , and flux ϕ . For convenience, we classify it into three categories: (1) Thermally induced (high T , low ϕ), (2) Irradiation induced (low T , high ϕ), (3) Thermoirradiation induced (high T , high ϕ). One-dimensional constitutive relations for each of these categories of creep are developed here by various physical arguments based on experimental observation.

2(a). Thermally induced creep

Many models for thermal creep are given in [2], resulting in a variety of strain-time relations which contain the effects of temperature and stress. Following the approach in [3], we write the strain ϵ , at time t , for constant temperature T_0 and constant stress σ_0 , as a sum of linear elastic, thermally induced steady creep, thermally induced transient creep, and thermal expansion components, i.e.,

$$\epsilon = \frac{\sigma_0}{E} + F_T^{(1)}(\sigma_0)\Phi_T(T_0)t + \sum_{i=2}^M F_T^{(i)}(\sigma_0)[1 - \exp(-A_T^{(i)}\Phi_T(T_0)t)] + \alpha_T\theta_0 \quad (1)$$

Here, E is the elastic modulus, $F_T^{(1)}$ are functions of stress, Φ_T is a function of temperature, $A_T^{(1)}$ are constants, and α_T is the coefficient of thermal expansion for temperature increment, $\theta_0 = T_0 - T_r$, relative to constant reference temperature T_r . In the remainder of this section, we first generalize Equation (1) to the case of variable stress and then generalize it to the case of variable temperature.

Generalization to Variable Stress $\sigma(t)$

The first term in Equation (1), the elastic strain, is, of course, already valid for variable stress $\sigma(t)$. Various techniques may be employed to extend the second and third terms to the case of variable $\sigma(t)$. Following the strain hardening approach in [3], we assume that each component of

creep strain $\epsilon^{(k)}$ may be written as a function of stress and strain, i.e.,

$$\frac{\partial \epsilon^{(k)}}{\partial t} = f[\sigma(t), \epsilon^{(k)}(t)] \quad (2)$$

Applying this approach to Equation (1) term by term, we obtain

$$\frac{\partial \epsilon_s(t)}{\partial t} = F_T^{(i)}(\sigma(t)) \Phi_T(T_0) \quad (3)$$

$$\frac{\partial \epsilon_t^{(i)}}{\partial t} + A_T^{(i)} \Phi_T(T_0) \epsilon_t^{(i)} = A_T^{(i)} \Phi_T(T_0) F_T^{(i)}(\sigma(t)) \quad i = 2, \dots, M \quad (4)$$

where ϵ_s is the steady creep and $\epsilon_t^{(i)}$ is the i -th component of transient creep. With the use of an integration factor and an integration by parts, an integral form of Equation (1) valid for variable stress may be obtained as

$$\begin{aligned} \epsilon = \frac{\sigma}{E} &+ \int_0^t F_T^{(i)}(\sigma) \Phi_T(T_0) dt' + \sum_{i=2}^M A_T^{(i)} \Phi_T(T_0) \exp[-A_T^{(i)} \Phi_T(T_0)t] \\ &\cdot \int_0^t \exp[A_T^{(i)} \Phi_T(T_0)t'] F_T^{(i)}(\sigma) dt' + d_T \theta_0 \end{aligned} \quad (5)$$

Equation (5) may also be rewritten in terms of memory integrals as

$$\begin{aligned} \epsilon = \frac{\sigma}{E} &+ \int_0^t \Phi_T(T_0)(t-t') \frac{\partial F_T^{(i)}(\sigma)}{\partial t'} dt' \\ &+ \sum_{i=2}^M \int_0^t \{ 1 - \exp[-A_T^{(i)}(T_0)(t-t')] \} \frac{\partial F_T^{(i)}(\sigma)}{\partial t'} dt' \\ &+ d_T \theta_0. \end{aligned} \quad (6)$$

The integrals in Eq. (6) are modified superposition integrals similar to the type first introduced by Leaderman.

Generalization to Variable Temperature T(t)

We assume that the temperature dependence of the material properties is of the thermorheologically simple type. As in [4], we introduce a reduced time as

$$\xi_T = \int_0^t \Phi_T(T(t')) dt' \quad (7)$$

and transform real time space (x_i, t) into (x_i, ξ_T) space. For clarity, we place a carat above each dependent variable in the new space, for example

$$\hat{f}(x_i, \xi_T) \equiv f(x_i, t) = f(x_i, g^{-1}(x_i, \xi_T)) \quad (8)$$

where $g^{-1}(x_i, \xi_T)$ is the inverse of Equation (7).

Accordingly, Equation (6) becomes

$$\begin{aligned} \hat{\varepsilon} &= \frac{\hat{\sigma}}{E} + \int_0^{\xi_T} (\xi_T - \xi'_T) \frac{\partial \hat{F}_T^{(i)}(\hat{\sigma})}{\partial \xi'_T} d\xi'_T \\ &+ \sum_{i=2}^M \int_0^{\xi_T} \left\{ 1 - \exp[-A_T^{(i)}(\xi_T - \xi'_T)] \right\} \frac{\partial \hat{F}_T^{(i)}(\hat{\sigma})}{\partial \xi'_T} d\xi'_T \\ &+ d_T \hat{\theta} \end{aligned} \quad (9)$$

Finally, Equation (9) may be rewritten as

$$\hat{\varepsilon} = \frac{\hat{\sigma}}{E} + \sum_{i=1}^M \int_0^{\xi_T} \hat{J}_T^{(i)}(\xi_T - \xi'_T) \frac{\partial \hat{F}_T^{(i)}(\hat{\sigma})}{\partial \xi'_T} d\xi'_T + d_T \hat{\theta} \quad (10)$$

where $J_T^{(i)}(\xi_T)$ are the thermal steady and transient creep compliance functions $J_T^{(i)}(\xi_T) = \xi_T$ and $J_T^{(i)}(\xi_T) = 1 - \exp(-A_T^{(i)}\xi_T)$, $i = 2, \dots, M$.

Equation (10) may be represented by a non-linear generalized Kelvin model, and the terms in the summation are modified superposition integrals for a thermorheologically simple material. In the case where aging effects are significant, a time hardening procedure may be used to generate creep compliances $J_T^{(i)}(\xi_T, \xi'_T)$ as in [3]; aging effects are also discussed in detail by Krempl [21].

2(b). Irradiation Induced Creep

For emphasis, we summarize here some pertinent earlier remarks on irradiation creep given in the introduction. Some data has been obtained for steels and alloys by various constant stress and flux tests, such as the stretched

spring [5], pressurized tube [6], bent beam [7], torsion rod [8] and uniaxial tension [9], as summarized in [19]. Although reliable data are still scarce (especially at high fluences), a reasonable model can be constructed. It is found that the expression for irradiation induced strain at constant flux (ϕ_0) and constant stress (σ_0) is similar to Equation (1). However, the material creep parameters now depend on a function of the flux rather than of the temperature, and the thermal expansion term in Equation (1) is replaced by a swelling term. It is also found that, in the absence of aging (i.e. annealing), the flux is proportional to the rate of change of the point defect density \dot{n} . For the convenience of our thermodynamic development, we choose to express the material creep parameters as functions of \dot{n} rather than ϕ . The swelling term is often less significant than the other strain terms, and we express it simply as linear in $\phi_0 t$. Since ϕ_0 is proportional to \dot{n}_0 , we may rewrite this term as linear in the imperfection density increment $v_0 = n_0 - n_r$ relative to the reference n_r . Accordingly,

$$\begin{aligned}\epsilon &= \frac{\sigma_0}{E} + F_R^{(1)}(\sigma_0) \Psi_R(\dot{n}_0) t \\ &+ \sum_{i=2}^N F_R^{(i)}(\sigma_0) [1 - \exp(-A_R^{(i)} \Psi_R(\dot{n}_0) t)] + \alpha_R v_0\end{aligned}\quad (11)$$

where $F_R^{(i)}$ are functions of stress, Ψ_R is a function of \dot{n}_0 (i.e. flux), $A_R^{(i)}$ are constants and α_R is the coefficient of swelling. Equation (11) is remarkably similar to Equation (1), but in addition to the replacement of T_0 by \dot{n}_0 and θ_0 by v_0 , it differs in two important respects: (1) Whereas the $F_T^{(1)}(\sigma_0)$ are highly non-linear, the $F_R^{(i)}(\sigma_0)$ are often chosen linear, (2) While ϕ_T is an exponential-like function of temperature, Ψ_R is often linear in \dot{n} (i.e. the flux).

Proceeding by analogy with thermally induced creep, we may generalize each irradiation induced creep term to variable stress by the strain hardening technique (no aging), and then to variable $\dot{\eta}$ by introducing an irradiation reduced time

$$\xi_R = \int_0^t \Psi_R(\dot{\eta}(t')) dt' \quad (12)$$

Thus,

$$\hat{\epsilon} = \frac{\hat{\sigma}}{E} + \sum_{i=1}^N \int_0^{\xi_R} J_R^{(i)}(\xi_R - \xi'_R) \frac{\partial F_R^{(i)}(\hat{\sigma})}{\partial \xi'_R} d\xi'_R + \alpha_R \hat{\nu} \quad (13)$$

where $J_R^{(i)}$ are the irradiation induced creep compliance functions. Continuing the analogy with thermally induced creep, we designate the material as fluxorheologically simple. Note that, for all $F_R^{(i)}(\sigma)$ linear, the summation in Equation (13) may be dropped if a summed compliance is introduced. Finally, we note that aging may be more significant in irradiation creep than in thermally induced creep, and thus it may be necessary to employ a time hardening procedure to generate the irradiation creep compliance functions $J_R^{(i)}(\xi_R, \xi'_R)$.

2 (c). Thermoirradiation Induced Creep

At high temperature and high neutron flux, the constitutive relation will likely be some complicated extension and combination of Equations (10) and (13). Very little is known about this relation at present, but it appears that the thermally induced creep parameters will now also depend on some weak function of flux $\Psi_T(\dot{\eta})$ i.e., irradiation enhancement). For example, in [22], the thermally induced creep rate is increased a factor Ψ_T equal to $(1 + \beta\dot{\eta})$. Similarly, the irradiation induced creep parameters will depend on some weak function of temperature $\Phi_R(T)$. An example of this is given in [23], where a function $\Phi_R(T)$ is given as $\exp(1.405 - 0.0027T)$. Thus, the thermal and irradiation reduced times ξ_T and ξ_R are now replaced respectively by

$$\eta_T = \int_0^t \Phi_T(T(t')) \Psi_T(\dot{\eta}(t')) dt' \quad \text{and} \quad \eta_R = \int_0^t \Phi_R(T(t')) \Psi_R(\dot{\eta}(t')) dt' \quad (14)$$

Utilizing these replacements in Equations (10) and (13), and assuming that the results may be superposed, we obtain

$$\begin{aligned} \hat{\varepsilon} = & \frac{\hat{\sigma}}{E} + \sum_{i=1}^M \int_0^t J_T^{(i)} (\eta_T - \eta'_T) \frac{\partial \hat{F}_T^{(i)}(\hat{\sigma})}{\partial \eta'_T} d\eta'_T \\ & + \sum_{i=1}^N \int_0^t J_R^{(i)} (\eta_R - \eta'_R) \frac{\partial \hat{F}_R^{(i)}(\hat{\sigma})}{\partial \eta'_R} d\eta'_R + \alpha_T \hat{\theta} + \alpha_R \hat{\nu} \end{aligned} \quad (15)$$

The material characterized by this relation will be termed thermofluxorheologically simple. In some cases, it may also be necessary to generalize the thermal expansion and swelling terms through the introduction of coupled functions such as $G_T(v, \theta)$. There appears to be no experimental evidence suggesting that the thermal expansion term $\alpha_T \theta$ be modified to include a dependence on v . However, temperature may enter into the swelling term as in [24], where the factor $\exp\left\{-\left[\frac{Q_1}{RT}\right] - \left[\frac{Q_2}{RT}\right]\right\}$ is introduced. In this paper, we assume that α_T and α_R are independent of θ and v .

Converting Equation (15) into real time space, we obtain the final result

$$\begin{aligned} \varepsilon = & \frac{\sigma}{E} + \sum_{i=1}^M \int_0^t J_T^{(i)} (\eta_T(t) - \eta_T(t')) \frac{\partial F_T^{(i)}(\sigma)}{\partial t'} dt' \\ & + \sum_{i=1}^N \int_0^t J_R^{(i)} (\eta_R(t) - \eta_R(t')) \frac{\partial F_R^{(i)}(\sigma)}{\partial t'} dt' \\ & + \alpha_T \theta + \alpha_R \nu \end{aligned} \quad (16)$$

This result differs from that given in [10] in two respects: (1) Two physically different reduced time scales are present rather than one, and (2) The nonlinear stress dependence is obtained by generalizing stress to

functions of stress rather than by introducing stress into the compliance.

We now seek to show that Equation (16) is consistent with a thermodynamic development of three-dimensional constitutive relations.

3. GENERAL MEMORY INTEGRAL

In the absence of neutron irradiation, the first law of thermodynamics for infinitesimal theory (e.g. see [15]) is given by the equation

$$-\varepsilon_{ij} \frac{\partial \sigma_{ij}}{\partial t} - \frac{\partial Q_i}{\partial x_i} = \rho \left(-\frac{\partial G}{\partial t} + s \frac{\partial T}{\partial t} + T \frac{\partial s}{\partial t} \right) \quad (17)$$

Here, ε_{ij} is the strain tensor, σ_{ij} the stress tensor, $\partial Q_i / \partial x_i$ the divergence of heat efflux vector Q_i , ρ the mass density (assumed constant), G the Gibbs free energy ($G = \sigma_{ij} \varepsilon_{ij} - u + Ts$, with σ_{ij} and T as independent variables), s the entropy and u the internal energy.

When a metal is exposed to neutron flux, additional energy is transferred throughout the mass (by collision) primarily into two effects: (1) The generation of local heating via lattice vibrations, and (2) The production of point defects. The rate of energy addition via the first effect may be expressed simply by the volume integral $\int_V r dv$, where r is the strength of a distributed internal heat source [25]. In considering the second effect, it seems reasonable to postulate that the rate of energy added per unit volume in the creation of point defects is proportional to erf_A inspection density. Thus, we may express the rate of energy addition due to the second effect as

$$\dot{R} = - \int_V e \frac{\partial n}{\partial t} dv \quad (18)$$

where the negative sign is taken for convenience of later development and e is a material property (the conjugate property to n).

The global form of the first law of thermodynamics is now given as

$$\dot{P} + \dot{Q} + \dot{R} = \dot{U} \quad (19)$$

Here, in addition to the usual rate of work (\dot{P}) and time rate of change of total internal energy (\dot{U}) terms, there is the rate of heat influx term (\dot{Q})

as modified by the addition of a heat source component, and the new term \dot{R} as previously defined in Equation (18). For infinitesimal theory, the \dot{P} , \dot{Q} and \dot{U} terms may be written as

$$\dot{P} = \int_V F_i v_i dV + \int_V \frac{\partial v_i}{\partial x_i} \sigma_{ij} dV + \int_V v_i \frac{\partial \sigma_{ij}}{\partial x_i} dV \quad (20a)$$

$$\dot{Q} = - \int_V \frac{\partial Q_i}{\partial x_i} dV + \int_V \rho r dV \quad (20b)$$

$$\dot{U} = \frac{\partial}{\partial t} \left[\int_V \frac{1}{2} \rho v_i v_i dV + \int_V \rho u dV \right] \quad (20c)$$

where F_i is the body force and v_i is the velocity. Combining Equations (18), (19), (20) and the definition of G in the usual manner, we obtain the extended version of local Equation (17) as

$$-\varepsilon_{ij} \frac{\partial \sigma_{ij}}{\partial t} - \frac{\partial Q_i}{\partial x_i} - \rho r - e \frac{\partial n}{\partial t} = \rho \left(-\frac{\partial G}{\partial t} - s \frac{\partial T}{\partial t} - T \frac{\partial s}{\partial t} \right) \quad (21)$$

where now the independent variables for G are σ_{ij} , T and n . The second law is expressed by the usual Clausius-Duhem inequality with a heat source term

$$\rho T \frac{\partial s}{\partial t} - \rho r + \frac{\partial Q_i}{\partial x_i} - \frac{Q_i}{T} \frac{\partial T}{\partial x_i} \geq 0 \quad (22)$$

Combining Equations (21) and (22), and introducing v and θ as previously defined, we obtain for infinitesimal theory

$$-\varepsilon_{ij} \frac{\partial \sigma_{ij}}{\partial t} - \rho \frac{\partial G}{\partial t} - \rho s \frac{\partial \theta}{\partial t} - \frac{Q_i}{T} \frac{\partial \theta}{\partial t} - e \frac{\partial v}{\partial t} \geq 0 \quad (23)$$

This inequality forms the basis of the development in this section, where now the independent variables are σ_{ij} , θ and v .

Proceeding in a manner similar to that in [15], as modified by the addition of the new independent variable v , we express the Gibbs free

energy as

$$PG = G^I[\sigma_{ij}(x_i, t), \theta(x_i, t), v(x_i, t)] + G^M[\sigma_{ij}'(x_i, t), \theta(x_i, t), v(x_i, t), t] \quad (24)$$

where G^I is the Gibbs free energy function (per unit volume) due to the instantaneous response, and G^M the Gibbs free energy function due to memory. We postulate for G^M the sum of nonlinear memory integrals

$$G^M = \sum_{r=1}^P f_{mn}^{(r)}(\sigma_{ij}, \theta, v) \int_0^t g_{kl}^{(r)}(\sigma_{ij}', \theta', v') \frac{\partial J_{mnkl}^{(r)}(\eta^{(r)}(t) - \eta^{(r)}(t'))}{\partial(t-t')} dt' \quad (25)$$

where $f_{mn}^{(r)}$, $g_{kl}^{(r)}$ are tensor functions of σ_{ij} , θ and v , and the $J_{mnkl}^{(r)}$ are time dependent material properties.

Applying Leibnitz's rule to Equations (24) and (25), we obtain the time derivative

$$\begin{aligned} P \frac{\partial G}{\partial t} &= \frac{\partial G^I}{\partial \sigma_{ij}} \frac{\partial \sigma_{ij}}{\partial t} + \frac{\partial G^I}{\partial \theta} \frac{\partial \theta}{\partial t} + \frac{\partial G^I}{\partial v} \frac{\partial v}{\partial t} \\ &+ \sum_{r=1}^P \left[\frac{\partial f_{mn}^{(r)}}{\partial \sigma_{ij}} \frac{\partial \sigma_{ij}}{\partial t} + \frac{\partial f_{mn}^{(r)}}{\partial \theta} \frac{\partial \theta}{\partial t} + \frac{\partial f_{mn}^{(r)}}{\partial v} \frac{\partial v}{\partial t} \right] \int_0^t g_{kl}^{(r)} \frac{\partial J_{mnkl}^{(r)}(\eta^{(r)}(t) - \eta^{(r)}(t'))}{\partial(t-t')} dt' \\ &+ \sum_{r=1}^P f_{mn}^{(r)} g_{kl}^{(r)} \left[\frac{\partial J_{mnkl}^{(r)}(\eta^{(r)}(t) - \eta^{(r)}(t'))}{\partial(t-t')} \right]_{t=t} + \sum_{r=1}^P f_{mn}^{(r)} \int_0^t g_{kl}^{(r)} \frac{\partial^2 J_{mnkl}^{(r)}(\eta^{(r)}(t) - \eta^{(r)}(t'))}{\partial(t-t') \partial t} dt' \end{aligned} \quad (26)$$

We now substitute Equation (26) into inequality (23) and group terms to obtain

$$\begin{aligned} &\left\{ -E_{ij} + \frac{\partial G^I}{\partial \sigma_{ij}} + \sum_{r=1}^P \frac{\partial f_{mn}^{(r)}}{\partial \sigma_{ij}} \int_0^t g_{kl}^{(r)} \frac{\partial J_{mnkl}^{(r)}(\eta^{(r)}(t) - \eta^{(r)}(t'))}{\partial(t-t')} dt' \right\} \frac{\partial \sigma_{ij}}{\partial t} \\ &+ \left\{ -Ps + \frac{\partial G^I}{\partial \theta} + \sum_{r=1}^P \frac{\partial f_{mn}^{(r)}}{\partial \theta} \int_0^t g_{kl}^{(r)} \frac{\partial J_{mnkl}^{(r)}(\eta^{(r)}(t) - \eta^{(r)}(t'))}{\partial(t-t')} dt' \right\} \frac{\partial \theta}{\partial t} \\ &+ \left\{ -e + \frac{\partial G^I}{\partial v} + \sum_{r=1}^P \frac{\partial f_{mn}^{(r)}}{\partial v} \int_0^t g_{kl}^{(r)} \frac{\partial J_{mnkl}^{(r)}(\eta^{(r)}(t) - \eta^{(r)}(t'))}{\partial(t-t')} dt' \right\} \frac{\partial v}{\partial t} \\ &+ \sum_{r=1}^P f_{mn}^{(r)} g_{kl}^{(r)} \left[\frac{\partial J_{mnkl}^{(r)}(\eta^{(r)}(t) - \eta^{(r)}(t'))}{\partial(t-t')} \right]_{t=t} \\ &+ \sum_{r=1}^P f_{mn}^{(r)} \int_0^t g_{kl}^{(r)} \frac{\partial^2 J_{mnkl}^{(r)}(\eta^{(r)}(t) - \eta^{(r)}(t'))}{\partial(t-t') \partial t} dt' - \frac{Q_i}{T_r} \frac{\partial \theta}{\partial t} \geq 0 \end{aligned} \quad (27)$$

Inequality (27) must hold for arbitrary values of $\partial u / \partial t$, $\partial \sigma_{ij} / \partial t$ and $\partial \theta / \partial t$; this then implies the constitutive relations

$$E_{ij} = \frac{\partial G^I}{\partial \sigma_{ij}} + \sum_{r=1}^P \frac{\partial f_{mn}^{(r)}}{\partial \sigma_{ij}} \int_0^t g_{kl}^{(r)} \frac{\partial J_{mnkl}^{(r)}(\eta^r(t) - \eta^r(t'))}{\partial(t-t')} dt' \quad (28a)$$

$$\rho_s = \frac{\partial G^I}{\partial \theta} + \sum_{r=1}^P \frac{\partial f_{mn}^{(r)}}{\partial \theta} \int_0^t g_{kl}^{(r)} \frac{\partial J_{mnkl}^{(r)}(\eta^r(t) - \eta^r(t'))}{\partial(t-t')} dt' \quad (28b)$$

$$e = \frac{\partial G^I}{\partial v} + \sum_{r=1}^P \frac{\partial f_{mn}^{(r)}}{\partial v} \int_0^t g_{kl}^{(r)} \frac{\partial J_{mnkl}^{(r)}(\eta^r(t) - \eta^r(t'))}{\partial(t-t')} dt' \quad (28c)$$

As a consequence of Equation (28), inequality (27) now requires

$$\Lambda - \frac{Q_i}{T_r} \frac{\partial \theta}{\partial x_i} \geq 0 \quad (29)$$

where Λ is the dissipation function given by

$$\Lambda = \sum_{r=1}^P f_{mn}^{(r)} g_{kl}^{(r)} \left[\frac{\partial J_{mnkl}^{(r)}(\eta^r(t) - \eta^r(t'))}{\partial(t-t')} \right]_{t=t'} + \sum_{r=1}^P f_{mn}^{(r)} \int_0^t g_{kl}^{(r)} \frac{\partial^2 J_{mnkl}^{(r)}(\eta^r(t) - \eta^r(t'))}{\partial(t-t') \partial t} dt' \quad (30)$$

Since Inequality (29) must be satisfied for any case, we may consider the special case of uniform temperature field $\frac{\partial \theta}{\partial x_i} = 0$, which then requires

$$\Lambda \geq 0 \quad (31)$$

Returning again to arbitrary $\frac{\partial \theta}{\partial x_i}$, we obtain from (29) and (31)

$$\frac{Q_i}{T_r} \frac{\partial \theta}{\partial x_i} \geq 0 \quad (32)$$

This result is consistent with the Fourier heat conduction law

$$Q_i = -k_{ij} \frac{\partial \theta}{\partial x_j} \quad (33)$$

and was obtained in a similar manner by Christensen in [26].

With the use of Equations (24), (25) and (28), coupled energy equation (21) may now be rewritten for infinitesimal theory as

$$\rho r - \frac{\partial Q_i}{\partial x_i} + \Lambda - T_r \frac{\partial}{\partial t} \left[\frac{\partial G^I}{\partial \theta} + \sum_{m=1}^P \frac{\partial f_{mn}^{(r)}}{\partial \theta} \int_0^t g_{mn}^{(r)} \frac{\partial J_{mnk}^{(r)} (\gamma^m(t) - \gamma^m(t'))}{\partial (t-t')} dt \right] = 0 \quad (34)$$

where the term in the brackets is the entropy s . A coupled energy equation for linearized infinitesimal theory may be obtained by dropping the second order term Λ .

In the following sections, we will show that superposition integral (16) is in fact a special case of Equation (28 a).

4. MODIFIED SUPERPOSITION INTEGRAL

In this section, we obtain a three-dimensional constitutive equation containing superposition integrals similar to those in Equation (16). To this end, we will set in Equations (28)

$$P = M + N \quad (35a)$$

$$f_{mn}^{(r)} = \sigma_{mn} \quad (35b)$$

$$g_{kl}^{(r)} = \begin{cases} (F_T)_{kl}^{(r)}(\sigma_{ij}) & r = 1, 2, \dots, M \\ (F_R)_{kl}^{(r)}(\sigma_{ij}) & r = M+1, \dots, M+N \end{cases} \quad (35c)$$

$$J_{mnkl}^{(r)} = \begin{cases} (J_T)_{mnkl}^{(r)}(\eta_T(t)) & r = 1, 2, \dots, M \\ (J_R)_{mnkl}^{(r)}(\eta_R(t)) & r = M+1, \dots, M+N \end{cases} \quad (35d)$$

$$\text{and } G^x = \bar{G} + \bar{\epsilon}_{ij} \sigma_{ij} + p \bar{s} \theta + \bar{e} \nu + \frac{1}{2} C_{ijkl} \sigma_{ij} \sigma_{kl} + \frac{1}{2} \frac{C_G}{T_r} \theta^2 + \frac{1}{2} p \nu^2 + (\alpha_T)_{ij} \sigma_{ij} \theta + h \theta \nu + (\alpha_R)_{ij} \nu \sigma_{ij} + \sum_{r=1}^M (J_T)_{ijkl}^{(r)}(0) \int_0^t (F_T)_{kl}^{(r)}(\sigma'_{ij}) d\sigma'_{ij} + \sum_{r=1}^N (J_R)_{ijkl}^{(r)}(0) \int_0^t (F_R)_{kl}^{(r)}(\sigma'_{ij}) d\sigma'_{ij} \quad (35e)$$

Here $(F_T)_{mn}^{(r)}$ and $(F_R)_{mn}^{(r)}$ are functions of stress only, C_{ijkl} is the linear elastic compliance, $(\alpha_T)_{ij}$ the thermal expansion coefficient, C_G the specific heat at constant stress, \bar{G} , $\bar{\epsilon}_{ij}$, \bar{s} and \bar{e} are initial values, and p , h and $(\alpha_R)_{ij}$ are new material constants.

Employing Equations (35a-e) in Equations (28) we obtain

$$\begin{aligned} \epsilon_{ij} = & C_{ijkl} \sigma_{kl} + (\alpha_T)_{ij} \theta + (\alpha_R)_{ij} \nu + \sum_{r=1}^M \int_0^t (J_T)_{ijkl}^{(r)}(\eta_T(t) - \eta_T(t')) \frac{\partial (F_T)_{kl}^{(r)}(\sigma'_{ij})}{\partial t'} dt' \\ & + \sum_{r=1}^N \int_0^t (J_R)_{ijkl}^{(r)}(\eta_R(t) - \eta_R(t')) \frac{\partial (F_R)_{kl}^{(r)}(\sigma'_{ij})}{\partial t'} dt' \end{aligned} \quad (36a)$$

$$\rho s = \rho \bar{s} + (\alpha_T)_{ij} \sigma_{ij} + \frac{\rho C_p}{T_r} \theta + h\nu \quad (36b)$$

$$e = \bar{e} + (\alpha_R)_{ij} \sigma_{ij} + h\theta + p\nu \quad (36c)$$

where we have used an integration by parts and the conditions $\sigma_{ij}(0^-) = \epsilon_{ij}(0^-) = 0$. Inserting the entropy (36b) into energy equation (34), we also obtain

$$\rho r - \frac{\partial Q_i}{\partial x_i} + \Lambda - \rho C_p \frac{\partial \theta}{\partial t} - T_r (\alpha_T)_{ij} \frac{\partial \sigma_{ij}}{\partial t} - T_r h \frac{\partial \nu}{\partial t} = 0 \quad (37)$$

Equation (36a) is clearly the desired extension of constitutive equation (16) to three dimensions. Entropy expression (36b) is the same as in linear thermoelasticity, with the addition of the structure dependent term $h\nu$. Finally, if we drop the second order dissipation term from coupled energy equation (37), we obtain the result of linear thermoelasticity supplemented by the two new energy terms ρr and $-T_r h \frac{\partial \nu}{\partial t}$.

For an isotropic material, Fourier heat conduction equation (33) reduces to

$$Q_i = -k \frac{\partial \theta}{\partial x_i} \quad (38)$$

and then energy equation (37) becomes

$$\rho r + k \frac{\partial^2 \theta}{\partial x_i^2} + \Lambda - \rho C_p \frac{\partial \theta}{\partial t} - T_r (\alpha_T)_{ij} \frac{\partial \sigma_{ij}}{\partial t} - T_r h \frac{\partial \nu}{\partial t} = 0 \quad (39)$$

Finally, the stress-strain relation (36a) is written for an isotropic material as

$$\epsilon_{ij} = \frac{1+\nu_e}{E} \sigma_{ij} - \frac{\nu_e}{E} \sigma_{kk} \delta_{ij} + \alpha_T \delta_{ij} \theta + \alpha_R \delta_{ij} \nu$$

$$\begin{aligned} & + \sum_{r=1}^M \int_0^t \left\{ (J_{T_I}^{(r)}(\eta_T(t) - \eta_T(t')) \frac{\partial (F_T')^{(r)}_{ij}}{\partial t'} + \frac{1}{3} [(J_T^{(r)}(\eta_T(t) - \eta_T(t')) - (J_T^{(r)}(\eta_T(t) - \eta_T(t')))] \frac{\partial (F_T')^{(r)}_{kk}}{\partial t'} \delta_{ij} \right\} dt' \\ & + \sum_{r=1}^N \int_0^t \left\{ (J_{R_I}^{(r)}(\eta_R(t) - \eta_R(t')) \frac{\partial (F_R')^{(r)}_{ij}}{\partial t'} + \frac{1}{3} [(J_R^{(r)}(\eta_R(t) - \eta_R(t')) - (J_R^{(r)}(\eta_R(t) - \eta_R(t')))] \frac{\partial (F_R')^{(r)}_{kk}}{\partial t'} \delta_{ij} \right\} dt' \quad (40) \end{aligned}$$

Here, ν_e and E are the elastic Poisson's ratio and Young's modulus respectively, and the subscripts I and II designate shear and bulk creep compliance functions respectively. In the next section we discussed the power law form of Equation (40).

5. ISOTROPIC STRESS POWER LAW

In [27], a nonlinear constitutive equation of the isotropic power law type was developed for a three-dimensional viscoelastic material with instantaneous and time dependent compressibility. The development was based on the assumption that creep potentials may be obtained by raising the elastic quadratic form to integer powers, and furthermore the creep compliances used were for the generalized Kelvin model. In [15], Chang et al. equated the integrated function in the Gibbs free energy (in our case $g_{ij} = (F_T)_{ij}$ or $(F_R)_{ij}$ in Equations (25, 36a, 40)) to the derivative $(\frac{\partial}{\partial \sigma_{ij}})$ of the above mentioned potentials, and obtained the same single integral representation. In this section we shall also use these same forms for the functions $(F_T)_{ij}$ and $(F_R)_{ij}$ in Equation (40), and furthermore we will also utilize generalized Kelvin compliances.

Proceeding as described above, we obtain the isotropic stress power law

$$\begin{aligned} \epsilon_{ij} = & \frac{1+\nu_e}{E} (\sigma_{ij} - \frac{\nu_e}{1+\nu_e} I_1 \delta_{ij}) + \alpha_T \theta \delta_{ij} + \alpha_R \nu \delta_{ij} \\ & + C_{T_S} \int_0^t J_{T_S} (\gamma_T(t) - \gamma_T(t')) \frac{\partial}{\partial t'} \left\{ \left[J_2 + \frac{1-2\nu_{T_S}}{6(1+\nu_{T_S})} I_1^2 \right] (\sigma_{ij} - \frac{\nu_{T_S}}{1+\nu_{T_S}} I_1 \delta_{ij}) \right\} dt' \\ & + \sum_{k=2}^M C_{T_k}^{(k)} \int_0^t J_{T_k}^{(k)} (\gamma_T(t) - \gamma_T(t')) \frac{\partial}{\partial t'} \left\{ \left[J_2 + \frac{1-2\nu_{T_k}^{(k)}}{6(1+\nu_{T_k}^{(k)})} I_1^2 \right] (\sigma_{ij} - \frac{\nu_{T_k}^{(k)}}{1+\nu_{T_k}^{(k)}} I_1 \delta_{ij}) \right\} dt' \\ & + C_{R_S} \int_0^t J_{R_S} (\gamma_R(t) - \gamma_R(t')) \frac{\partial}{\partial t'} \left\{ \left[J_2 + \frac{1-2\nu_{R_S}}{6(1+\nu_{R_S})} I_1^2 \right] (\sigma_{ij} - \frac{\nu_{R_S}}{1+\nu_{R_S}} I_1 \delta_{ij}) \right\} dt' \\ & + \sum_{k=2}^N C_{R_k}^{(k)} \int_0^t J_{R_k}^{(k)} (\gamma_R(t) - \gamma_R(t')) \frac{\partial}{\partial t'} \left\{ \left[J_2 + \frac{1-2\nu_{R_k}^{(k)}}{6(1+\nu_{R_k}^{(k)})} I_1^2 \right] (\sigma_{ij} - \frac{\nu_{R_k}^{(k)}}{1+\nu_{R_k}^{(k)}} I_1 \delta_{ij}) \right\} dt' \end{aligned}$$

(41)

In the above, $I_1 = \sigma_{kk}$ is the first invariant of stress and $J_2 = 1/2 s_{ij} s_{ij}$ is the second invariant of the stress deviator $s_{ij} = \sigma_{ij} - 1/3 \sigma_{kk} \delta_{ij}$. Subscripts Ts, Tt, Rs and Rt designate thermal steady, thermal transient, irradiation steady and irradiation transient creep quantities respectively. Accordingly, the various creep compliances are defined as

$$\begin{aligned} J_{Ts}(\gamma_T(t)) &= \gamma_T(t) \\ J_{Rs}(\gamma_R(t)) &= \gamma_R(t) \\ J_{Tt}^{(k)}(\gamma_T(t)) &= 1 - \exp[-A_T^{(k)} \gamma_T(t)] \quad k = 2, \dots, M \\ J_{Rt}^{(k)}(\gamma_R(t)) &= 1 - \exp[-A_R^{(k)} \gamma_R(t)] \quad k = 2, \dots, N \end{aligned} \tag{42}$$

in conformity with Equations (1) and (11). The powers $m_{Ts}^{(k)}$, $m_{Tt}^{(k)}$, $m_{Rs}^{(k)}$ and $m_{Rt}^{(k)}$ are all restricted to integers 0, 1, 2, ...; the constants $v_{Ts}^{(k)}$, $v_{Tt}^{(k)}$, $v_{Rt}^{(k)}$, $v_{Rs}^{(k)}$ are Poisson's coefficients for the various creep components; and the $C_{Ts}^{(k)}$, $C_{Tt}^{(k)}$, $C_{Rs}^{(k)}$ and $C_{Rt}^{(k)}$ are creep constants which multiply the various compliances.

The time dependent compressibility is displayed by forming the contraction of Equation (41)

$$\begin{aligned} \epsilon_{ll} &= \frac{1-2\nu_e}{E} \sigma_{ll} + \frac{(1-2\nu_{Ts})C_{Ts}}{1+\nu_{Ts}} \int_0^t J_{Ts}(\gamma_T(t)-\gamma_T(t')) \frac{\partial}{\partial t'} \left\{ [J_2 + \frac{1-2\nu_{Ts}}{6(1+\nu_{Ts})} I^2] (\sigma_{ll}) \right\} dt' \\ &\quad + \sum_{k=2}^M \frac{(1-2\nu_{Tt}^{(k)})C_{Tt}^{(k)}}{1+\nu_{Tt}^{(k)}} \int_0^t J_{Tt}^{(k)}(\gamma_T(t)-\gamma_T(t')) \frac{\partial}{\partial t'} \left\{ [J_2 + \frac{1-2\nu_{Tt}^{(k)}}{6(1+\nu_{Tt}^{(k)})} I^2] (\sigma_{ll}) \right\} dt' \\ &\quad + \frac{(1-2\nu_{Rs})C_{Rs}}{1+\nu_{Rs}} \int_0^t J_{Rs}(\gamma_R(t)-\gamma_R(t')) \frac{\partial}{\partial t'} \left\{ [J_2 + \frac{1-2\nu_{Rs}}{6(1+\nu_{Rs})} I^2] (\sigma_{ll}) \right\} dt' \\ &\quad + \sum_{k=2}^N \frac{(1-2\nu_{Rt}^{(k)})C_{Rt}^{(k)}}{1+\nu_{Rt}^{(k)}} \int_0^t J_{Rt}^{(k)}(\gamma_R(t)-\gamma_R(t')) \frac{\partial}{\partial t'} \left\{ [J_2 + \frac{1-2\nu_{Rt}^{(k)}}{6(1+\nu_{Rt}^{(k)})} I^2] (\sigma_{ll}) \right\} dt' \\ &\quad + 3\alpha_R \nu + 3\alpha_T \theta \end{aligned} \tag{43}$$

This equation is useful in analyzing pressure test data.

For the one-dimensional creep test, $\sigma_{xx} = \sigma_0 H(t)$ where σ_0 is constant and $H(t)$ is the unit step function, and the flux and temperature are also kept constant at ϕ_0 and T_0 . Equation (41) then yields

$$\begin{aligned} \varepsilon_{xx} = & \left\{ \frac{\sigma_0}{E} + \left(\frac{\sigma_0}{\lambda_T} \right)^{n_T} \Phi_T \Psi_T t + \sum_{k=2}^M \left(\frac{\sigma_0}{\mu_T^{(k)}} \right)^{q_T^{(k)}} [1 - \exp(-t A_T^{(k)} \Phi_T \Psi_T)] + \left(\frac{\sigma_0}{\lambda_R} \right)^{n_R} \Phi_R \Psi_R t \right. \\ & \left. + \sum_{k=2}^N \left(\frac{\sigma_0}{\mu_R^{(k)}} \right)^{q_R^{(k)}} [1 - \exp(-t A_R^{(k)} \Phi_R \Psi_R)] \right\} H(t) + d_T \theta_0 + d_R \nu_0 \end{aligned} \quad (44a)$$

$$\begin{aligned} \varepsilon_{yy} = \varepsilon_{zz} = & - \left\{ \nu_e \frac{\sigma_0}{E} + \nu_s \left(\frac{\sigma_0}{\lambda_T} \right)^{n_T} \Phi_T \Psi_T t + \sum_{k=2}^M \nu_T^{(k)} \left(\frac{\sigma_0}{\mu_T^{(k)}} \right)^{q_T^{(k)}} [1 - \exp(-t A_T^{(k)} \Phi_T \Psi_T)] \right. \\ & + \nu_{Rs} \left(\frac{\sigma_0}{\lambda_R} \right)^{n_R} \Phi_R \Psi_R t + \sum_{k=2}^N \nu_{Rt}^{(k)} \left(\frac{\sigma_0}{\mu_R^{(k)}} \right)^{q_R^{(k)}} [1 - \exp(-t A_R^{(k)} \Phi_R \Psi_R)] \left. \right\} H(t) + d_T \theta_0 + d_R \nu_0 \end{aligned} \quad (44b)$$

In Equations (44) we have eliminated reduced times n_T and n_R by means of Equation (14) with constant ϕ_T , ψ_T , ϕ_R and ψ_R , and we have also introduced

$$n_T = 2m_{Tb} + 1 \quad \lambda_T = \frac{2^{m_{Tb}/(2m_{Tb}+1)} (m_{Tb}+1)/(2m_{Tb}+1)}{(C_{Tb})^{1/(2m_{Tb}+1)}}$$

$$q_T^{(k)} = 2m_{Tb}^{(k)} + 1 \quad \mu_T^{(k)} = \frac{2^{m_{Tb}^{(k)}/(2m_{Tb}^{(k)}+1)} (m_{Tb}^{(k)}+1)/(2m_{Tb}^{(k)}+1)}{(C_{Tb}^{(k)})^{1/(2m_{Tb}^{(k)}+1)}}$$

$$n_R = 2m_{Rb} + 1 \quad \lambda_R = \frac{2^{m_{Rb}/(2m_{Rb}+1)} (m_{Rb}+1)/(2m_{Rb}+1)}{(C_{Rb})^{1/(2m_{Rb}+1)}}$$

$$q_R^{(k)} = 2m_{Rb}^{(k)} + 1 \quad \mu_R^{(k)} = \frac{2^{m_{Rb}^{(k)}/(2m_{Rb}^{(k)}+1)} (m_{Rb}^{(k)}+1)/(2m_{Rb}^{(k)}+1)}{(C_{Rb}^{(k)})^{1/(2m_{Rb}^{(k)}+1)}} \quad (45)$$

The role of the various Poisson's coefficients becomes clear when we compare Equation (44b) with (44a). In [28] it is suggested that ν_{Ts} is close to 1/2 while $\nu_{Tt}^{(k)}$ lie somewhere between ν_e and 1/2. Although some information on ν_{Rs} is given in [19], there appears to be no data available on $\nu_{Rt}^{(k)}$. On comparing Equation (44a) with Equations (1) and (11), we see that we have essentially obtained our initial equations. As remarked earlier, irradiation creep is usually taken as linear in stress. Thus we may, with little loss of accuracy, set $n_R = q_R^{(k)} = 1$ in Equations (44). Finally, one may readily specialize energy equation (39) for the isotropic power law by evaluating dissipation function (30) for this material, but as previously noted it is common practice to simply drop the dissipation function from the energy equation.

6. SUMMARY

One-dimensional constitutive equations valid for thermal and irradiation creep at constant stress were generalized to variable stress by utilizing the strain hardening technique to obtain modified superposition integrals. Thermal creep terms valid for variable temperature were obtained first by assuming a thermorheologically simple material, ^{and} accordingly introducing a thermal reduced time to account for temperature dependence of the material parameters. Similarly, irradiation creep ^{terms} valid for variable flux were obtained by assuming a fluxorheologically simple material, and introducing an irradiation reduced time dependent on the ^{time} rate of change of the imperfection density. An expression for thermoirradiation creep (i.e., temperature and flux both high) was then obtained by superposition of the previous thermal and irradiation creep terms, as modified by a redefinition of the two reduced time scales to account for coupling. Linear elastic, thermal expansion and swelling terms were also included in the one-dimensional constitutive equation.

A systematic development based on the first and second laws of thermodynamics then enabled us to obtain general memory integrals which contain as a special case those previously obtained in the paper by physical argument. The conventional forms of the infinitesimal laws of thermodynamics were first modified to account for the effect of neutron flux on local heating and point defect production. A Gibbs free energy was assumed as the sum of an instantaneous function component and a memory functional component with stress, temperature increment and defect density increment as independent variables. As a result, three-dimensional anisotropic constitutive equations were obtained for the strain, entropy and

a material property conjugate to the defect density; the coupled energy equation, dissipation function and heat conduction law were also obtained.

Modified superposition integrals similar to those previously developed were then obtained by substituting special functions into the results of the thermodynamic analysis. Finally, an isotropic stress power law was obtained through the use of quadratic forms raised to integer powers, and was then examined for three-dimensional time dependent compressibility and one-dimensional Poisson contraction.

The validity of the analogy between thermally induced and irradiation induced creep as presented here must of course be tested against further experimental evidence. However, we have conclusively shown that the analogy as deduced from simple one-dimensional data is consistent with a more general thermodynamic development. We also point out that the analogy should prove useful in reducing the complexity of future testing programs.

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Langley Research Center
Langley Station
Hampton, Virginia 23365
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National Aeronautic & Space Admin.
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PART 2 - CONTRACTORS AND OTHER
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Division of Engineering
Brown University
Providence, Rhode Island 02912

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Dept. of Engr. Mechs.
University of Alabama
Huntsville, Alabama 35804

Professor R.S. Rivlin
Center for the Application of Mathematics
Lehigh University
Bethlehem, Pennsylvania 18015

Professor Julius Miklowitz
Division of Engr. & Applied Sciences
California Institute of Technology
Pasadena, California 91109

Professor George Sih
Department of Mechanics
Lehigh University
Bethlehem, Pennsylvania 18015

Dr. Harold Liebowitz, Dean
School of Engrg. & Applied Science
George Washington University
725 23rd Street
Washington, D.C. 20006

Professor Eli Sternberg
Division of Engr. & Applied Sciences
California Institute of Technology
Pasadena, California 91109

Professor Burt Paul
University of Pennsylvania
Towne School of Civil and Mechanical Engrg.
Room 113 Towne Building
220 S. 33rd Street
Philadelphia, Penna. 19104

Professor S.B. Dong
University of California
Department of Mechanics
Los Angeles, California 90024

Professor Paul M. Maghdci
Div. of Applied Mechanics
Etchegerry Hall
University of California
Berkeley, California 94720

Professor W. Nachbar
University of California
Department of Aerospace & Mech. Engrg
La Jolla, California 92037

Professor J. Baltrukonis
Mechanics Division
The Catholic Univ. of America
Washington, D.C. 20017

Professor A.J. Durelli
Mechanics Division
The Catholic Univ. of America
Washington, D.C. 20017

Professor H.H. Bleich
Department of Civil Engrg.
Columbia University
Amsterdam & 120th Street
New York, New York 10027

Professor R.D. Mindlin
Department of Civil Engrg.
Columbia University
S.W. Mudd Building
New York, New York 10027

Professor A.M. Freudenthal.
George Washington University
School of Engrg. & Applied Science
Washington, D.C. 20006

Professor B.A. Boley
Department of Theor. & Appl. Mech.
Cornell University
Ithaca, New York 14850

Professor P.G. Hodge
Department of Mechanics
Illinois Institute of Technology
Chicago, Illinois 60616

Dr. D.C. Drucker
Dean of Engineering
University of Illinois
Urbana, Illinois 61801

Professor N.M. Newmark
Dept. of Civil Engineering
University of Illinois
Urbana, Illinois 61801

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Dept. of Engrg. Mechs.
North Carolina State University
Raleigh, North Carolina 27607

Dr. George Herrmann
Stanford University
Department of Applied Mechanics
Stanford, California 94305

Professor J.D. Achenbach
Technological Institute
Northwestern University
Evanston, Illinois 60201

Director, Ordnance Research Lab.
Pennsylvania State University
P.O. Box 30
State College, Pennsylvania 16801

Professor J. Klemperer
Dept. of Aero. Engrg. & Applied Mech.
Polytechnic Institute of Brooklyn
333 Jay Street
Brooklyn, New York 11201

Professor J. Klosner
Polytechnic Institute of Brooklyn
333 Jay Street
Brooklyn, New York 11201

Professor A.C. Eringen
Dept. of Aerospace & Mech. Sciences
Princeton University
Princeton, New Jersey 08540

Dr. S.L. Koh
School of Aero., Astro. & Engr. Sc.
Purdue University
Lafayett, Indiana 47907

Professor R.A. Schapery
Civil Engineering Department
Texas A & M University
College Station, Texas 77840

Professor E.H. Lee
Div. of Engrg. Mechanics
Stanford University
Stanford, California 94305

Dr. Nicholas J. Hoff
Dept. of Aero. & Astro.
Stanford University
Stanford, California 94305

Professor Max Anliker
Dept. of Aero. & Astro.
Stanford University
Stanford, California 94305

Professor Chi-Chang Chao
Div. of Engr. Mechanics
Stanford University
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Professor J.E. Fitzgerald, Ph.
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University of Utah
Salt Lake City, Utah 84122

Professor R.J.H. Bell
Chairman, Aeronautical Engr. Dept.
207 Guggenheim Hall
University of Washington
Seattle, Washington 98105

Professor Albert S. Kobayashi
Dept. of Mechanical Engr.
University of Washington
Seattle, Washington 98105

Professor G.R. Irwin
Dept. of Mech. Engrg.
Lehigh University
Bethlehem, Pennsylvania 18015

Dr. Daniel Frederick
Dept. of Engr. Mechs.
Virginia Polytechnic Inst.
Blacksburg, Virginia 24061

Professor Lambert Tall
Lehigh University
Department of Civil Engrg.
Bethlehem, Pennsylvania 18015

Professor M.P. Wnuk
South Dakota State University
Department of Mechanical Engineering
Brookings, South Dakota 57006

Professor Norman Jones
Massachusetts Institute of Technology
Dept. of Naval Architecture & Marine Engrg.
Cambridge, Massachusetts 02139

Professor Pedro V. Marcal
Brown University
Division of Engineering
Providence, Rhode Island 02912

Professor Werner Goldsmith
Department of Mechanical Engineering
Division of Applied Mechanics
University of California
Berkeley, California 94720

Professor R.B. Testa
Dept. of Civil Engineering
Columbia University
S.W. Mudd Bldg.
New York, New York 10027

Dr. Y. Weitsman
Dept. of Engng. Sciences
Tel-Aviv University
Ramat-Aviv
Tel-Aviv, Israel

Professor W.D. Pilkey
Dept. of Aerospace Engr.
University of Virginia
Charlottesville, Virginia 22903

Professor W. Prager
Division of Engineering
Brown University
Providence, Rhode Island 02912

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Mr. Carl E. Hartbower
Dept. 4620, Bldg. 2019 A2
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Director
Ship Research Institute
Ministry of Transportation
700, SHINKAWA
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Tokyo, Japan

Dr. H.N. Abramson
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78206

Dr. R.C. DeHart
Southwest Research Institute
6500 Culebra Road
San Antonio, Texas 78206

Mr. Roger Weiss
High Temp. Structures & Materials
Applied Physics Lab.
8621 Georgia Ave.
Silver Spring, Maryland 20910

Mr. E.C. Francis, Head
Mech. Props. Eval.
United Technology Center
Sunnyvale, California 94088

Mr. C.N. Robinson
Atlantic Research Corp.
Shirley Highway at Edsall Road
Alexandria, Virginia 22231

Mr. P.C. Durup
Aeromechanics Dept., 74-43
Lockheed-California Co.
Burbank, California 91503

Mr. D. Wilson
Litton Systems, Inc.
AMTD, Dept. 400
El Segundo
9920 W. Jefferson Blvd.
Culver City, California 90230

Dr. Kevin J. Forsberg, Head
Solid Mechanics
Orgn 52-20, Bldg. 205
Lockheed Palo Alto Research Lab.
Palo Alto, California 94302

Dr. E.M.Q. Roren
Head, Research Department
Det Norske Veritas
Post Box 6060
Oslo, Norway

Dr. Andrew F. Conn
Hydronautics, Incorporated
Pindell School Road, Howard County
Laurel, Maryland 20810